behavior moving in response to the electric field. Oscillations in a system such as this are similar to those encountered in shock structure in a plasma.[13]

Generally, the effects of charge on the particles in the relaxation zone is small. For many cases of interest the effect on the gas motion is negligible as long as the loading and charge is kept low. For drag correlations other than Stokes drag law and particles larger than a micron the particle motions are only slightly affected. These facts suggest that the electric field generated by the shock might be utilized as a diagnostic technique, although it must be remembered that charge and size of the particle will be complicated distribution.

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Optimum Design of Damped Vibration Absorbers Over a Finite Frequency Range

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Introduction

A GREAT deal of attention has recently been given to application of optimization theory and techniques to a variety of engineering design problems involving dynamic response of mechanical systems. Hamad[1] considered optimum design of a dynamic absorber, using a direct search technique for the solution. Wilmert and Fox[2] have considered optimum design of a class of linear, multi-degree of freedom shock isolation systems. Schmit and Fox[3] and Schmit and Rybicki[4] treated dynamic response optimization of a simple shock isolation system, using a steepest descent method with alternate steps. In this paper, optimum design of a dynamic vibration absorber is treated by an optimal design formulation that enforces performance constraints over a range of excitation frequencies. A steepest descent programing technique is used to solve the resulting optimal design problem.[5,6]

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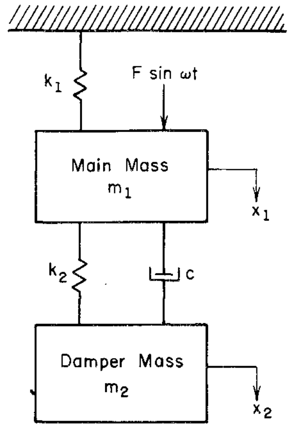

Fig. 1 Dynamic absorber.

Optimal Design Formulation

In many practical situations, the main mass of a system may undergo large amplitude vibration, especially when the exciting frequency is close to the resonant frequency of the system. There are many techniques of reducing the amplitude of these vibrations,[7] one of which is to attach a secondary mass system to the main mass. This secondary mass system is known as the absorber system. The main mass of the system is subjected to a forcing function of frequency ω.

The dynamic absorber considered is shown in Fig. 1. The analysis of dynamic behavior of this system can be found in many textbooks.[7] The notation used in the statement and analysis of the problem is defined as follows: $x_{st} = F/k_1$ is the static deflection of the main mass, produced by force F, $\Omega_n = (k_1/m_1)^{1/2}$ is the uncoupled natural frequency of the main system, $\omega_n = (k_2/m_2)^{1/2}$ is the uncoupled natural frequency of the damper system, $\mu = m_2/m_1$ is the ratio of the masses of the absorber and the main system, $f = \omega_n/\Omega_n$ is the ratio of the uncoupled natural frequencies of the absorber and the main mass, $g = \omega/\Omega_n$ is the ratio of the exciting frequency to the uncoupled natural frequency of the main mass, $C_c = 2m_2\Omega_n$ is the critical damping, c = damping coefficient, $\xi = c/C_c$ is the damping ratio, $x_1 = x_1(\xi, f, g)$ is extreme displacement of mass 1, and $x_2 = x_2(\xi, f, g)$ is extreme displacement of mass 2. Two optimal design problems for this system can be defined as follows.

Problem 1: For a given g, find the ξ and f that minimize the ratio of extreme displacement of the main mass to its static displacement

$$J_1 = x_1(\xi, f, g)/x_{st} \tag{1}$$

subject to the "rattle space" and extreme value design variable constraints

$$\left|\frac{x_2 - x_1}{x_1}\right| \leqq Q_{\max} \tag{2}$$

$$\xi_{\min} \leqq \xi \leqq \xi_{\max} \tag{3}$$

and

$$f_{\min} \leqq f \leqq f_{\max} \tag{4}$$

This is a relatively straightforward nonlinear programing problem that may be solved by any of a number of techniques.

Hamad used an exhaustive search technique to obtain the solution for this problem.[1] He simply divided the design space into a finite mesh and checked each point of the grid for a minimum point. In the present paper a generalized steepest descent method is used to obtain an optimum solution of the problem.[5,6]

Problem 2 is similar to Problem 1 in that it seeks to minimize the response of the main mass of the system, but over a range of excitation frequency ratio, g. The "rattle space" constraint must, therefore, be satisfied over a full range of allowed excitation frequency. Mathematically, this may be stated as follows: find ξ and f that minimize

$$J_2 = \max_{a \leq g \leq b} [x_1(\xi, f, g)/x_{st}] \tag{5}$$

subject to the design variable constraint Eqs. (3) and (4), and

$$\max_{a \leq g \leq b} \left| \frac{x_2(\xi, f, g) - x_1(\xi, f, g)}{x_1(\xi, f, g)} \right| \leq Q_{\max} \tag{6}$$

The solution of this minimax need not be a saddle point, so no minimax numerical technique can be routinely applied.[8,9] In the present paper, this problem is treated in an approximate way. The forcing frequency g is discretized and the "rattle space" constraint is imposed at each point of the grid. An upper bound of the objective function over $a \leq g \leq b$ is denoted as an artificial design variable d, i.e., $x_1(\xi, f, g)/x_{st} \leq d$. Let g_i, $i = 1, \ldots, m$, denote a selected grid of points in the closed interval $[a, b]$. The minimax problem can then be treated as a nonlinear programing problem as follows: Find ξ, f, and $d \geq 0$ that minimize the upper bound d, subject to constraint Eqs. (3) and (4),

$$x_1(\xi, f, g_i)/x_{st} \leq d \qquad i = 1, \ldots, m \tag{7}$$

and

$$\left| \frac{x_2(\xi, f, g_i) - x_1(\xi, f, g_i)}{x_1(\xi, f, g_i)} \right| \leq Q_{\max} \qquad i = 1, \ldots, m \tag{8}$$

In this discretized formulation, there are $2m+2$ constraints. A generalized steepest descent method, introduced in Refs. 5 and 6, is used to solve this problem. In this iterative technique, all the constraints of the problem are checked for violation at each design cycle and only the constraints that are violated are retained for the subsequent sensitivity analysis. Since the active constraint set is generally small, the computational effort at each design cycle is quite small.

Numerical Results

The two problems of the preceding section were solved, using Hamad's data[1] for comparison purposes: $\mu = 0.3$, $\xi_{\min} = 0.0$, $\xi_{\max} = [3\mu/8(1+\mu)^3]^{1/2}$, $f_{\min} = 0.0$, $f_{\max} = 2.0$, and $Q_{\max} = 3.0$. Problem 1 was solved for excitation frequency ratios of 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, and 1.5. Results obtained herein and by Hamad[1] are shown in Fig. 2. The present method gave substantially lower values of the objective

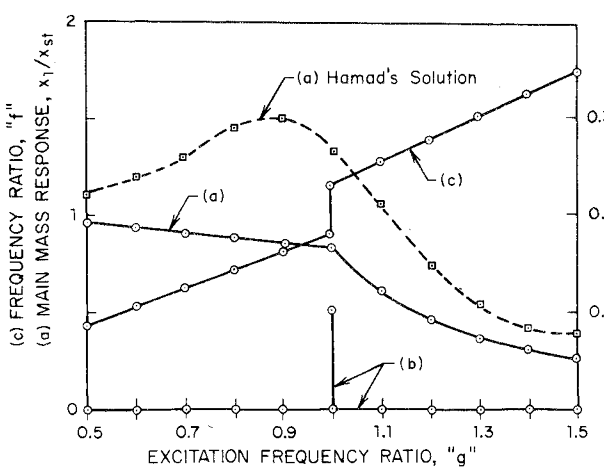

Fig. 2 Optimum isolator designs for Problem 1.

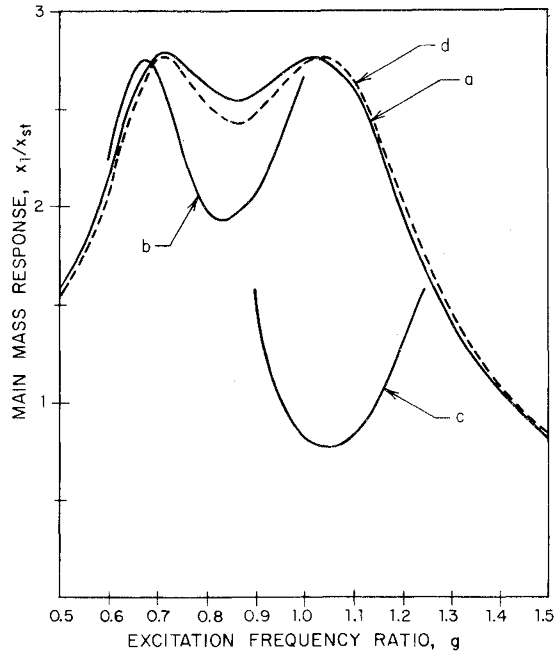

Fig. 3 Optimum isolator designs for Problem 2.

function than those obtained by Hamad.[1] It is interesting to note that the optimum solutions are undamped for all values of g, except at the singularity $g = 1.0$, where an infinite number of solutions exist. It can be shown that the optimum values of ξ and f are independent of the mass ratio.[10]

Problem 2 was solved with three different ranges of excitation frequency ratio. Figure 3 summarizes the main mass response for each of the three sets of design constraints. The figure also contains Den Hartog's solution for an infinite range of excitation frequencies, obtained analytically in Ref. 7. The results in Fig. 3 are explained as follows:

a) Input data: $0 \leq \xi \leq 0.35$, $0 \leq f \leq 2.0$, $0.5 \leq g \leq 1.5$, grid size $= 0.05$.
Optimum results: $\xi = 0.2377$, $f = 0.7683$, $x_1/x_{st} = 2.7599$.

b) Input data: same as a) except, $0.6 \leq g \leq 1.0$.
Optimum results: $\xi = 0.1706$, $f = 0.7616$, $x_1/x_{st} = 2.6806$.

c) Input data: same as a) except, $0.9 \leq g \leq 1.25$.
Optimum results: $\xi = 0.1314$, $f = 1.0202$, $x_1/x_{st} = 1.578$.

d) Input data: same as a) except $0 \leq \xi \leq \infty$, $0 \leq f \leq \infty$, $0 \leq g \leq \infty$.
Den Hartog solution:[7] $\xi = 0.2263$, $f = 0.7692$, $x_1/x_{st} = 2.7736$.

From the study of Problem 2, the following observations may be made. 1) Curves a and d in Fig. 3 show that the approximate method used here compares quite well with the Den Hartog's solution for an infinite interval of excitation frequencies. 2) Curves b and c show that if one is concerned with a subset of the excitation interval, designs much superior to Den Hartog's may be found. 3) The formulation of the problem using artificial design variables and constraints has proven to be quite effective in computing a solution to the minimax problem.

Conclusions

Application of optimal design techniques to even the simplified problem treated in this paper indicates that minimax cost functions and maximum valued constraints, over a finite range of environmental parameters, can be effectively treated. The numerical results for Problem 2 show the dependence of the optimum design on the interval over which the free parameter may vary. If environmental controls narrow the interval of the free parameter, the optimum design is clearly improved. This is noted by curves b and c of Fig. 3.

Optimization of design for a fixed value of the free parameter in Problem 1, however, is impractical. Direct calculation of fixed frequency optimum isolator performance, for a frequency different from the given fixed frequency, shows that very large amplitude oscillations occur. It is necessary, therefore, to explicitly treat the variation of the free parameter within known or projected bounds.

The technique of treating the free parameter variation, by enforcing constraints and computing the cost function only at a finite grid on the free parameter domain appears to yield satisfactory results. There remains a need for research to develop a more efficient solution technique.

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Technical Comments

Comment on "Subsonic and Supersonic Boundary-Layer Flow Past a Wavy Wall"

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Introduction

RECENTLY Inger and Williams[1] investigated both theoretically and experimentally the problem of turbulent boundary-layer flow over a wave-shaped wall in the Mach number range of 0.8 to 1.8 at unit Reynolds number on the order of 10^6 per in. An essential feature of the inviscid part of Inger and Williams' study was the development of a "top-down" integration scheme whereby the two-point boundary value problem was converted into an "initial value" problem. The purpose of this Note is to point out that their calculation is only valid near the wall, and to show the appropriate process of converting into an initial value problem.

Analysis

The differential equation pertaining to the problem of a two-dimensional, steady compressible flow past a sinusoidal wavy wall of small amplitude has been derived before,[1,2] and it is in the form

$$\frac{d^2\tilde{p}}{dy^2} - 2\frac{dM/dy}{M}\frac{d\tilde{p}}{dy} + \alpha^2(M^2-1)\tilde{p} = 0 \qquad (1)$$

where $\tilde{p}$ is the complex variable of pressure perturbation, which is defined as $p = P_e + \tilde{p}(y)\, e^{i\alpha x}$, with $\alpha = 2\pi/\lambda$ the wave number, and $M(y)$ the meanflow local Mach number. The boundary conditions are[1,3]

$$d\tilde{p}/dy = -i(M_e{}^2-1)^{1/2}\alpha\tilde{p} \qquad \text{at} \qquad y = \delta \qquad (2)$$

and

$$d\tilde{p}/dy = -\varepsilon\alpha^2\rho u^2 \qquad \text{at} \qquad y = y_f \qquad (3)$$

where M_e is the outer-edge freestream Mach number, and y_f is a "cut-off" distance used in place of $y = 0$, since the solution is singular at $y = 0$ where $M \to 0$. The factor ε is the wave amplitude, ρ the mean flow density, and u the velocity. The outer-edge boundary condition, Eq. (2), implies that the corresponding disturbed pressure field behaves either like simple waves ($M_e > 1$) or exponentially decaying signals ($M_e < 1$). The inner boundary condition, Eq. (3), represents a kinematic tangency condition.

This is a two-point boundary value problem which can be solved by conventional "shooting" methods. However, instead of using the condition at the inner boundary, Eq. (3), Inger and Williams adopted the solution for uniform potential flow past a wavy wall as a second outer-edge boundary condition; namely, $\tilde{p} = \tilde{p}_{\text{potential}}$ at $y = \delta$. This converts the study into an initial value problem and is justified only if $\tilde{p}$ is properly scaled after the calculations.

To illustrate the appropriate conversion process, let $Q(y)$ be the solution of Eq. (1), which is defined by the boundary conditions

$$dQ/dy = -i(M_e{}^2-1)^{1/2}\alpha Q \qquad \text{at} \qquad y = \delta \qquad (4)$$

and

$$Q = Q_o \qquad \text{at} \qquad y = \delta \qquad (5)$$

It is easy to show, by virtue of the linearity of Eq. (1) and conditions of Eqs. (2) and (4), that $\tilde{p}$ is a simple multiple of the solution $Q(y)$. By condition of Eq. (3), the required multiple is

$$\tilde{p}(y) = -\left.\frac{\varepsilon\alpha^2\rho u^2}{dQ/dy}\right|_{y=y_f} Q(y) \qquad (6)$$

Note that, in general, Q_o can be any number, complex or real, and Eq. (6) will give the proper scaling that converts $Q(y)$ into $\tilde{p}(y)$, the expected physical answer. In our example, we set Q_o equal to the potential solution at $y = \delta$.

In the present investigation, the Adams-Bashforth-Moulton predictor-corrector scheme[4] is employed to integrate the differential equation across the boundary layer. The mesh size is automatically adjustable to stay within the accuracy criterion

$$\varepsilon_1 < \left|\frac{\tilde{p}_{\text{predictor}} - \tilde{p}_{\text{corrector}}}{\tilde{p}_{\text{corrector}}}\right| < \varepsilon_2$$

where ε_1 and ε_2 are assigned small quantities such as $\varepsilon_1 = 10^{-5}$ and $\varepsilon_2 = 10^{-2}$.

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